Topic 6-3: Thermal Conductivity
Kittel Pages: 121,122

Summary: We first develop an expression for heat flux of energy. We then convert this into physically meaningful terms and from this rearranging, gain an expression for thermal conductivity.

- Start with Fourier's law:

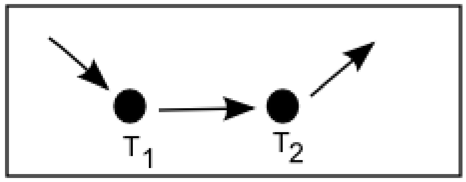
$$j_u = -\kappa \frac{dT}{dx} \qquad [1]$$

  - Related flux of thermal energy, j_u, to thermal conductivity, κ, and a temperature gradient
- **Approach:** Develop an expression for j_u, then identify the thermal conductivity, κ, term within this expression.
- Thermal conductivity is an intrinsic property
  - Independent of sample geometry (provided the sample is macro-scale)
- If you want to consider sample geometry use thermal conductance, $\frac{\kappa \cdot A}{L}$, where A is the corss-sectional area and L is the length
- Consider heat flux using ideal gas model
  - Have particles in a tube with a hot and cold side
- Think of flux in the positive x direction $= \frac{1}{2} n < |v_x| >$
  - One half term because half of the particles are going the wrong way
  - n= particle density
  - $< |v_x| >$ is the time averaged velocity component
- Want flux of energy not particles
- Let c be the heat capacity per particle
  - Motion from T to T+ΔT
- The particle starts at temperature T and moves in the temperature gradient to T+ΔT, where it absorbs energy cΔT as it thermalizes

$$j_{u_{T \to T+\Delta T}} = -\frac{1}{2} n < |v_x| > c\Delta T \qquad [2]$$

- Can get a net expression for the heat flux of $2(-\frac{1}{2} n < |v_x| > c\Delta T)$

- **Goal:** Convert this expression to physically useful terms (not just v_x and ΔT)
- Consider 2 thermalizing collisions



- We can describe ΔT in terms of a distance traveled between T1 and T2 in x (lD) and the associated temperature gradient: $\Delta T = T_2 - T_1 = l_x \frac{dT}{dx}$
- Recast length in terms of time and velocity: $\tau = \frac{l_x}{|v_x|}$
  - τ is the time between collisions
- Plugging in, we obtain: $\Delta T = \frac{dT}{dx}|v_x|\tau$
- Plugging in, we get the following for the heat flux:

$$j_{u_{T\to T+\Delta T}} = -n < |v_x| > c\frac{dT}{dx}|v_x|\tau = -n < {v_x}^2 > c\frac{dT}{dx}\tau \quad [3]$$

  - Recast $< {v_x}^2 >$ as the root mean squared velocity $v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2} = \sqrt{3{v_x}^2}$ assuming ${v_x}^2 = {v_y}^2 = {v_z}^2$
- Now ${v_x}^2 = \frac{1}{3}v^2$
- Again, plugging in:

$$j_{u_{T\to T+\Delta T}} = -\frac{1}{3}n < v^2 > c\frac{dT}{dx}\tau \quad [4]$$

  - Setting $v\tau = l$ and $nc = C$ where C is the heat capacity per unit volume, *l* is the mean free path between collisions, and *n* is the particle number density
  - we get

$$j_{u_{T\to T+\Delta T}} = -\frac{1}{3}Cvl\frac{dT}{dx} \quad [5]$$

- This means that $\kappa = \frac{1}{3}Cvl$
- Can obtain C and v from the dispersion relation!
- *l* comes from phonon scattering
  - If there is no scattering thermal conductivity goes to infinity

    - Scattering must be present in all materials, vibrations don’t infinitely propagate
  - Things that cause scattering
    - Non-infinite or non-perfect crystals
      - Point defects
      - Grain boundaries
      - Surfaces
    - Phonon-phonon scattering