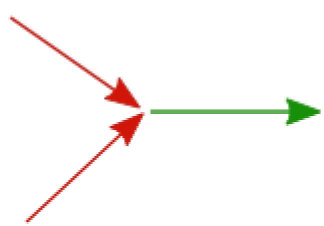
Topic 6-4: Phonon-Phonon Scattering
Kittel Pages: 123-126

Summary: First, we introduce scattering in terms of the familiar photons. Then we introduce two types of phonon scattering, the first being normal and the second being Umklapp scattering as well as the implications of each.

- Recall that a $\vec{q}$ outside of the first brillouin zone can be translated into the first brillouin zone by $\vec{G}$ with no loss of information (Nyquist frequency idea)
- For phonon-phonon scattering, we must remember (i) conservation laws, (ii) that phonons act as bosons and (iii) that phonons are quasi particles
    - Can create and destroy them

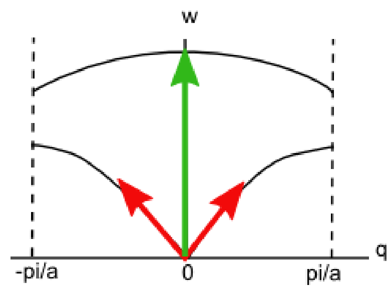
	Crystal Momentum	Energy
1 – Destruction	$\vec{q}_1 + \vec{q}_2 + \vec{G} = \vec{q}_3$	$E_1 + E_2 = E_3$
2 - Creation	$\vec{q}_1 = \vec{q}_2 + \vec{q}_3 + \vec{G}$	$E_1 = E_2 + E_3$

- In 1, we combine two phonons to create a third phonon
- In 2, we take a high energy phonon and split it into two other phonons
- Photon analogy: second harmonic generation
    - Red light creates green light

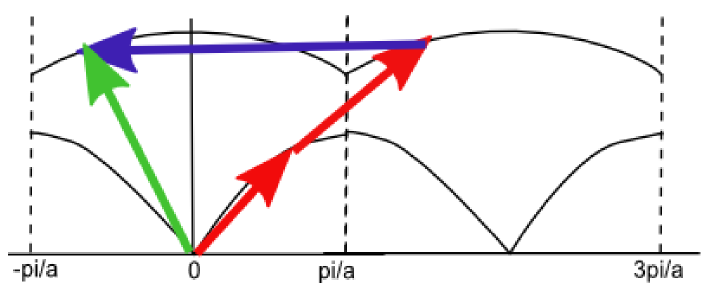


- Put 2 red photons into a crystal and there is a non-zero chance that these will combine to make green light
- 2 types of scattering
    - Normal: $\vec{G} = 0$ in crystal momentum conservation law, crystal momentum conserved.
    - Umklapp: $\vec{G} \neq 0$ (instead, it is a sum of the reciprocal lattice vectors) and crystal momentum is not conserved

- Normal phonon-phonon scattering



- 2 acoustical phonons (red) make an optical phonon (green)
- Not a scattering event that reduces thermal conductivity
- Does move system locally toward Planck distribution (i.e. if system phonon distribution is brought out of Plank distribution by an external source, normal phonon-phonon scattering will lead to a Planck distribution of phonon states)
- Transition must involve allowed states!
- Umklapp phonon-phonon scattering:



- $\vec{q}_1 + \vec{q}_2 + \vec{G} = \vec{q}_3\ (red + red + blue = green)$
    - $\vec{G} = -\vec{g}_1$
- Energy still conserved
- Crystal momentum not conserved
- The reverse process, the splitting case, is identical

- **Concept restatement:** Need to know impact of scattering on κ
- Scenario: Illuminate a surface with light such that you locally heat it and create a phonon source
- Cool the back side, so it is a thermal sink and a temperature gradient develops

- These optically-generated phonons will begin to diffuse from hot to cold
- In normal scattering, phonons redistribute but no net change in $\vec{q}$
  - No resistance to transporting heat
- With Umklapp scattering in this scenario, two phonons moving towards the cold side can combine and move back towards the hot side!
- Thinking of this scenario as the first few minutes of a marathon (where the runners are phonons moving from hot to col), but where participants merge together and start running back to the start (Umklapp scattering) is at least nightmare inducing, if not pedagogically useful.

Practical stuff:

- Umklapp scattering requires phonons to interact with each other. A large phonon population is thus required for Umklapp scattering to be a significant contributor to thermal conductivity
- As such, it will be the dominant source of phonon phonon scattering at room temperature and above, with a scattering rate proportional to population
- The mean free path between phonon-phonon scattering events thus scales as $l \propto \frac{1}{T}$
- Umklapp summary: failure to conserve crystal momentum due to anharmonic lattice which allows phonons to interact with one another and leads to significantly lower κ