Topic 8-2: Density of States and Fermi Dirac Distribution
Kittel Pages: 136-140

**Summary:** In this video we develop the density of states for electrons using the Fermi Dirac distribution. We then discuss how electrons fill states using the density of states expression and look at the Fermi Dirac distribution as temperature is increased.

- Developed a dispersion relation for a one electron system in previous video, $E = \frac{\hbar^2}{2m}|\vec{k}|^2$
- Now develop a density of states expression, D(E)
- Recall the density of states is found by $D(E) = \frac{dN}{dE}$
    - The number of electronic states within some dE is given by dN
    - N(E) is the number of states with energy less than or equal to E
- Already have an expression for N(E) of the traveling wave solution from phonons

$$N(E) = V_{sphere} \cdot \frac{1\ k\ point}{Volume\ per\ k\ point} \cdot 2 \quad [1]$$

- 2 comes from 2 spin orientations
- $V_{sphere} = \frac{4}{3}\pi|\vec{k}|^3$
- Volume per k point = $(\frac{2\pi}{L})^3$

- Need N(E) in terms of energy not k as it is above
- Can get this by solving for k in terms of E from the dispersion

$$V_{sphere} = \frac{4}{3}\pi|\vec{k}|^3 \quad [2]$$

$$E = \frac{\hbar^2}{2m}|\vec{k}|^2 \rightarrow |\vec{k}| = (\frac{2m}{\hbar}E)^{1/2} \quad [3]$$

$$V = \frac{4}{3}\pi(\frac{2mE}{\hbar^2})^{3/2} \quad [4]$$

$$N(E) = \frac{4}{3}\pi\left(\frac{2mE}{\hbar^2}\right)^{3/2}\left(\frac{1\ k\ point}{\frac{8\pi^3}{L^3}}\right)(2) \quad [5]$$

- Taking the derivative:

$$\frac{dN}{dE} = \frac{V}{2\pi^2}(\frac{2m}{\hbar})^{3/2}E^{1/2} = D(E) \quad [6]$$

- Normalizing for volume we get:

$$\frac{D(E)}{V} = \frac{E^{1/2}}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2} \quad [7]$$

- The right side of the expression is independent of sample volume
- We see that D(E) is proportional to $E^{1/2}$ in 3D; this is a good relation to keep in mind.

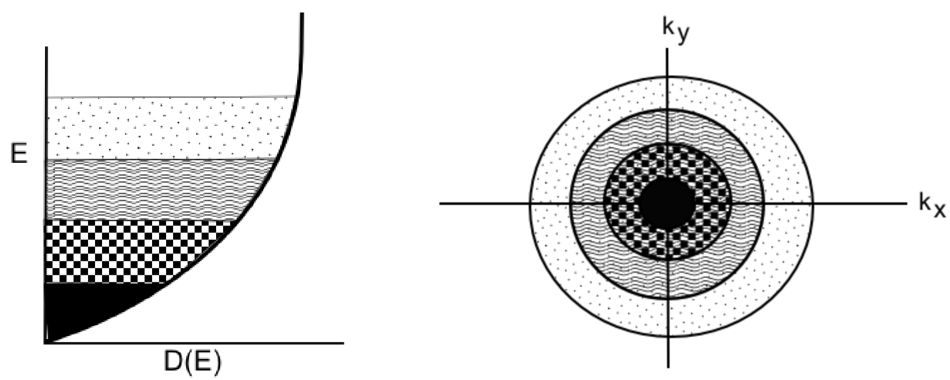
- Now use statistical thermodynamics to see how these states are filled as a function of temperature
- Imagine we have a Fermi gas at 0 K
  - System will have N electrons

$$N = \int_0^\infty dE\, D(E) f(E) \quad [8]$$

  - $f(E)$ is the occupation probability
- Instead of using the Planck distribution which is for bosons we will use the Fermi-Dirac distribution since electrons are fermions

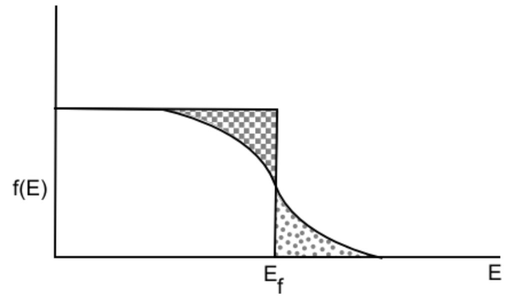
$$f(E) = \frac{1}{e^{(E-\mu)/k_B T}+1} \quad [9]$$

- Since we are dealing with fermions we can imagine our density of states as a massive bucket
- When we pour in electrons they will start by filling in the origin and will then move out
  - No two electrons can occupy the same state
- Since $D \propto E^{1/2}$ $\quad E \propto D^{1/2}$ so we can graph as below




- How high in energy the electrons fill this bucket is material dependent
  - Depends on the number of electrons per volume
- Define this maximum energy as the Fermi energy, $E_f$
- At 0 K $f(E) = 1\ if\ E \le E_f\ or\ 0\ if\ E > E_f$

  - This creates a perfect step function
- As we go up in temperature we see a smearing of this step function




- The polka dotted area us due to the excitation of electrons from thermal excitations
- The checker boarded area is the electrons that move up in these thermal excitations leaving behind empty modes

Questions to Ponder

1. Compare and contrast phonons in a harmonic lattice with the free electron model.